Design Charts for Vertical Drains Considering Soil Disturbance

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\section*{ABSTRACT}

The paper focuses on developing design charts that can be used to calculate, without iteration, the spacing of prefabricated vertical drains (PVDs) for a given soil condition and target degree of consolidation within a specified time. To this end, experimentally obtained hydraulic conductivity profiles in the disturbed zone surrounding the PVD are approximated by a sigmoidal function and incorporated in finite element analysis (FEA) for a parametric study. Subsequently, an analytical solution with bilinear variation of hydraulic conductivity in the disturbed zone is identified that produces PVD response comparable to those obtained from FEA. The analytical solution is used to develop design charts for a variety of conditions and hydraulic conductivity profiles that can be easily used in practice. The design charts can also be used for conditions in which overlapping of disturbed zones occurs.

\section*{INTRODUCTION}

Prefabricated vertical drains (PVDs) have been successfully used, mostly in conjunction with preloading, to improve the strength and stiffness of soft clayey deposits (Holtz 1987). PVDs act as vertical drainage channels that accelerate the consolidation of soft deposits; as water is expelled out of the ground, the void ratio of the soil decreases resulting in greater strength and stiffness. Notwithstanding their wide use, PVDs have certain operational problems. One such problem is the reduction of the soil hydraulic conductivity in the vicinity of PVDs due to disturbance caused by PVD installation. The reduced hydraulic conductivity in the zone of disturbance slows down the consolidation process.

Soil disturbance has been studied by many researchers and its effect has been accounted for in many analytical and experimental studies (Barron 1948, Hansbo 1981, Bergado et al. 1993, Basu and Prezzi 2007). Most of these research studies assumed that the reduced hydraulic conductivity in the zone of disturbance remains
spatially constant. However, recent experimental observations show that the hydraulic conductivity is not spatially constant within the zone of soil disturbance but increases gradually from a low value at the vicinity of the drain to the in situ value at the interface of the disturbed and undisturbed zones.

In this paper, we characterize and quantify the gradual variation of hydraulic conductivity in the disturbed zone and perform finite element analyses (FEA) to identify the important parameters that affect PVD performance. Subsequently, we identify an analytical solution based on bilinear variation of hydraulic conductivity in the disturbed zone that can be used in place of FEA for routine design calculations. Finally, we develop design charts based on the analytical solution that can be readily used without requiring any iteration for deciding PVD spacings.

DISTURBED ZONE CHARACTERIZATION

Recent experimental studies (Onoue et al. 1991, Madhav et al. 1993, Indraratna and Redana 1998, Sharma and Xiao 2000) show that, within the disturbed zone, the hydraulic conductivity varies spatially around PVDs. The spatial variation can be approximated by a sigmoidal curve (Figure 1):

\[
k_{hd} = k_{hs} + \left( 1 - \frac{k_{hs}}{k_{ho}} \right) \left( 1 - e^{-\frac{r}{r_{m,eq}}} \right)^\beta
\]

where \( k_{hd} \) is the hydraulic conductivity in the disturbed zone; \( k_{hs} \) is the hydraulic conductivity adjacent to the PVD; \( r \) is the radial distance from the PVD center (after converting the PVD cross-sectional area and the disturbed zone into equivalent circles), \( r_{m,eq} \) is the equivalent mandrel radius (obtained by equating the actual mandrel cross-sectional area to a circular area); and \( \alpha \) and \( \beta \) are fitting parameters. Hydraulic conductivity profiles with \( \alpha = 0.45-2.0 \) and \( \beta = 30 \) and with \( k_{hs}/k_{ho} = 0.1, 0.2 \) and 0.3 have been found to match the experimental data well (Figure 1).

The size and shape of the disturbed zone surrounding the PVD is not known with certainty. According to Figure 1, the location of the outer boundary of the disturbed zone (as measured from the center of the drain) can vary from 4 to 18 times the equivalent mandrel radius.

The shape of the disturbed zone is most likely governed by the mandrel shape. If a square or circular mandrel is used for PVD installation, the disturbed zone shape is likely to be a square or a circle in plan, while, for rectangular or diamond-shaped mandrels, it is likely to be an ellipse (Chai and Miura 1999). In our study, we consider both circular and elliptical disturbed zones. For the elliptical disturbed zone, we assume that the ratio of the semi-major axis \( a \) to the semi-minor axis \( b \) of the ellipse is the same as the aspect ratio \( a_{mn} = m_w/m_y \) of the mandrel cross section (\( m_w \) and \( m_y \) are the width and thickness of the mandrel cross section).

FINITE ELEMENT ANALYSIS

FEA of horizontal free-strain horizontal consolidation, following the Terzaghi-Rendulic theory of two-dimensional consolidation, were performed for an hexagonal and a square unit cell representing triangular and square PVD arrangements, respectively (Figure 2). PVDs with a typical cross section of 100 mm \( \times \) 4 mm were assumed for both arrangements with a center-to-center spacing equal to \( s \). The
hydraulic conductivity profiles assumed in the FEA follows those shown in Figure 1.

![Figure 1](image_url)

**Figure 1.** Observed and curve-fitted profiles of hydraulic conductivity.

A parametric study was performed using FEA (Basu and Prezzi 2009) and it was found that the degree of disturbance in the vicinity of the drain, quantified by the ratio $k_{hs}/k_{ho}$, the extent of the disturbed zone, the mandrel size, and PVD spacing have moderate to large effects on the consolidation rate. The degree of soil disturbance in the vicinity of the PVD (i.e., the $k_{hs}/k_{ho}$ ratio) and the extent of the disturbed zone are the two most important factors that significantly affect consolidation. Thus, it is important to properly establish the hydraulic conductivity profile in the field for accurate prediction of the consolidation rate.

**MATCHING OF FE SOLUTION WITH ANALYTICAL SOLUTION**

FEA are typically not done for routine PVD design; rather, the analytical
solution for PVD consolidation by Hansbo (1981), in which the PVD, the disturbed zone and the unit cell are converted into equivalent circles, is widely used. However, the solution by Hansbo (1981) assumes spatially constant hydraulic conductivity in the disturbed zone. No closed-form solution exists for sigmoidal variation of hydraulic conductivity in the disturbed zone. Basu et al. (2006) developed a series of closed-form solutions for PVD consolidation for different possible variations of hydraulic conductivity in the disturbed zone. One of the solutions, with bilinear variation of hydraulic conductivity, is used in place of the sigmoidal variation (Equation (1)) by matching the sigmoidal and the bilinear curves, as shown in Figure 3, and by converting the band-shaped PVD, elliptical-shaped disturbed zone and the square- or hexagonal-shaped unit cell into equivalent circles (the analytical solution was obtained for a circular drain with circular unit cell and a circular disturbed zone that has an inner smear zone and an outer transition zone; see Figure 3).

According to the analytical solution, the degree of consolidation $U$ is given by:

$$ U = 1 - e^{-\frac{8T}{\mu}} $$

(2)

where, $T = c_s t / 4 r_{c,eq}^2$ is the time factor ($c_s$ is the coefficient of consolidation for flow in the horizontal direction, and $r_{c,eq}$ is the radius of an equivalent circular unit cell which has the same area as the actual hexagonal or square unit cell; $r_{c,eq} = 0.525s$ and $0.564s$ for triangular and square PVD installation arrangements, respectively, where $s$ is the PVD spacing) and the parameter $\mu$ is equal to:

$$ \mu = n \left( \frac{n}{q} \right) + \frac{1}{\eta_s} \ln (m) + \frac{(p - m)}{(\eta_s p - \eta_s m)} \ln \left( \frac{\eta_s p}{\eta_s m} \right) + \frac{(q - p)}{(\eta_s q - p)} \ln \left( \frac{\eta_s q}{p} \right) - \frac{3}{4} $$

(3)

with $n > q > p > m$ and $\eta_p > \eta_s$. These parameters are defined as $n = r_e/r_d$, $m = r_s/r_d$, $q = r_e^2/r_d$, $p = r_p/r_d$, $\eta_s = k_{hs}/k_{ho}$ and $\eta_p = k_{hp}/k_{ho}$. In the foregoing equations, $r_d$, $r_s$, $r_t$ and $r_e$ are the radii of the drain, outer boundary of the smear zone, outer boundary of
the transition zone, and circular unit cell, respectively (Figure 3); \( r_p \) is the radius at which a change in slope occurs in the bilinear hydraulic conductivity profile; and \( k_{hp} \) is the hydraulic conductivity at \( r_p \). The equivalent circular drain radius \( r_{d,eq} \) for band-shaped PVDs is obtained by equating the perimeter of the PVD with that of the drain: 
\[
r_{d,eq} = \frac{d_w + d_t}{\pi},
\]
where \( d_w \) and \( d_t \) are the PVD width and thickness, respectively. The elliptical disturbed zone is converted into an equivalent circular zone (Figure 4). This conversion is reflected in the calculations of the equivalent circular radial distance \( r_{eq} \) for use in equation (1) given by 
\[
r_{eq} = \sqrt{\frac{x^2}{a_{dz}} + \frac{y^2}{a_{dz}}}.
\]
For matching the FEA with the analytical solutions, we assumed \( r_{c,eq} = r_c, r_{d,eq} = r_d, \) and \( r_{eq} = r \).

![Figure 4. Mapping of elliptical hydraulic conductivity contours into equivalent circles.](image)

The results obtained using the analytical solutions and the FEA were compared for square and triangular PVD arrangements, both with and without disturbance. In particular, we considered two cases: 1) spacing \( s = 1 \) m and a mandrel of 125 mm \( \times \) 50 mm and 2) spacing \( s = 3 \) m and a mandrel of 150 mm \( \times \) 150 mm. Profile 1 (\( \alpha = 2.0, \beta = 30 \) and \( k_{hs}/k_{ho} = 0.2 \)) of Figure 1 was chosen as the hydraulic conductivity profile for these two cases. The results of the FEA compare reasonably well with the analytical solutions. The difference in the estimation of \( T_{90} \) is approximately 7-17% between the analytical and finite element (FE) solutions.

The difference between the analytical and FE results occurs partly due to the difference in the shape of the unit cells and disturbed zones and partly due to the fact that equal-strain analytical solutions (where the vertical settlement is assumed uniform throughout the unit cell) and free-strain FE solutions (where no restriction on settlement is imposed) yield different results (Richart 1959). We observed that such a difference with FE results occurs also for the analytical solution of Hansbo (1981) (we performed FE analyses with only a smear zone and compared our results with the results of Hansbo (1981)). It is worth noting that actual consolidation in the field does not strictly follow the equal-strain or the free-strain condition, but is likely to be closer
to the equal-strain condition for closely spaced PVDs (Fox et al. 2005). The FE solutions have the advantage that the actual shape of the PVD, disturbed zone and unit cell can be simulated in the analysis. On the other hand, analytical solutions are more convenient to use in practice, and perhaps produce more accurate results for closely spaced PVDs.

**DESIGN CHARTS**

In the calculation of the PVD spacing required for a given soil condition using the analytical solution (equations (2) and (3)), an initial guess for the PVD spacing (which determines \( n \) in equation (3)) has to be made first, based on which the time factor corresponding to the desired degree of consolidation can be calculated. Consequently, iterations are required to determine the PVD spacing corresponding to the desired degree of consolidation within a certain time. Yeung (1997) developed a method by which this iteration procedure can be avoided. We adopt the method of Yeung (1997), and, as outlined below, apply it to the analytical solution.

A modified time factor \( T' \) can be defined as:

\[
T' = \frac{C_i t}{4r_d^2}
\]

Using the above definition, the degree of consolidation \( U \) in equation (2) can be rewritten as:

\[
U = 1 - e^{-\frac{8T'}{\mu}}
\]

where

\[
\mu' = n^2 \mu = n^2 \left[ \ln \left( \frac{n}{q} \right) + \frac{1}{\eta_s} \ln (m) + \frac{(p - m)}{(n_p - \eta_p m)} \ln \left( \frac{\eta_p}{\eta_p m} \right) + \frac{(q - p)}{(n_p q - p)} \ln \left( \frac{\eta_p q}{p} \right) - \frac{3}{4} \right]
\]

In the above equation for \( \mu' \), \( \mu \) is taken as equation (3) (note that \( n = r_e / r_d \)). If we normalize the new time factor \( T' \) with respect to \( \mu' \):

\[
\bar{T}' = T' / \mu'
\]

then \( \bar{T}' \) has a unique relationship with the degree of consolidation \( U \):

\[
\bar{T}' = -\frac{1}{8} \ln (1 - U)
\]

Equation (6) is used to develop curves relating \( n \) and \( \mu' \) for different hydraulic conductivity profiles. In order to develop the \( \mu' \) versus \( n \) charts, we assume that the hydraulic conductivity profiles in the field follow the trends shown in Figure 1. We choose three equivalent mandrel radius \( r_{m,eq} = 46.0 \text{ mm} \), 68.7 mm and 84.6 mm that are representative of the dimensions of the mandrels used in practice. We also assume PVD spacings within a 0.9-3.5 m range and a PVD dimension of 100 mm \( \times \) 4 mm \( (r_{d,eq} = 33.1 \text{ mm}) \), which gives \( n \) in the 15-60 range. Based on these inputs, we calculate the possible ranges for \( m, p, q, \eta_s \) and \( \eta_p \) (i.e., parameters related to the degree of disturbance and extent of the disturbed zone) for the different possible hydraulic conductivity profiles (Figure 1) and mandrel radii. Using these values, we generate the \( \mu' \) versus \( n \) charts, shown in Figures 5-7, for all the hydraulic
conductivity profiles. Following the procedure described above, site specific $\mu'$ versus $n$ curves can also be developed if the actual hydraulic conductivity profile is known.

In design, the drain geometry ($r_d$), soil properties ($c_h$), time available $t$ (within which a certain percentage of consolidation has to be completed) and the desired degree of consolidation $U$ are required as input. Using these inputs, $T'$ and $T''$ can be calculated from equations (4) and (8), respectively, from which $\mu'$ can be calculated using equation (7). With the calculated value of $\mu'$, $n$ can then be determined from the $\mu'$ versus $n$ charts (Figures 5–7), from which the desired PVD spacing can be calculated without any iteration.

Figure 5. $\mu'$ versus $n$ curves for $r_m = 46.0$ mm.

Figure 6. $\mu'$ versus $n$ curves for $r_m = 68.7$ mm.
OVERLAPPING OF DISTURBED ZONE

The hydraulic conductivity profiles considered in Figure 1 cause overlap of disturbed zones for some combinations of unit cell dimensions and mandrel size. Mathematically, the modified (overlapped) hydraulic conductivity profiles can be defined by the following ratios (Figure 8): \( l = r_l/r_d \) and \( \eta_l = k_{hl}/k_{ho} \) where \( r_l \) is the distance from the center of the drain to the point at which the zone of overlap starts; \( k_{hl} \) is the constant hydraulic conductivity in the overlap zone (we assume that the hydraulic conductivity remains constant at \( k_{hl} \) in the zone of overlap).

In order to produce \( \mu' \) versus \( n \) charts considering overlap, we assumed that the actual zones of overlap can be replaced by axisymmetric overlap zones (an assumption also made by Walker and Indraratna 2007). At the same time, modifications were required in the equation of \( \mu \) (equation (3)) in order to incorporate the overlap of disturbed zone (see Basu and Prezzi (2009) for details). The modified expressions of \( \mu \) were used in the present analysis and are incorporated in Figures 5–7, which can be used for cases with and without overlap of disturbed zones.
It is important to note that the enhanced reduction of hydraulic conductivity due to the overlap of disturbed zones reduces the consolidation rate more than what would occur without the overlap. Therefore, overlap of adjacent disturbed zones should be avoided in design.

NUMERICAL EXAMPLE

We work out a simple example problem to show how the design charts can be used in practice. A site with \( c_h = 2 \text{ m}^2/\text{year} \) is considered, where a 125 mm \( \times \) 50 mm mandrel (\( r_{m,eq} = 44.6 \text{ mm} \)) is to be used for installation of PVDs in a triangular pattern. The PVD cross section dimensions are 100 mm \( \times \) 4 mm (\( r_{d,eq} = 33.1 \text{ mm} \)). The hydraulic conductivity profile at the site is assumed to be profile 3 shown in Figure 1(a). It is required that 90% consolidation be attained within 1.5 years. Therefore, for \( U = 0.9 \), \( \bar{T}' \) can be obtained as 0.288 (equation (8)). Also, for \( r_{d,eq} = 33.1 \text{ mm} \), \( t = 1.5 \) years and \( c_h = 2 \text{ m}^2/\text{year} \), we get \( T' = 684.55 \) (equation (4)). Using \( \bar{T}' \) and \( T' \), we calculate \( \mu' = 2377 \) (equation (7)). Since the equivalent mandrel radius \( r_{m,eq} \) is close to 46.0 mm, we use Figure 5(a) (the curve corresponding to profile 3) to obtain \( n = 17.5 \). This gives \( r_{c,eq} = 0.58 \text{ m} \), from which we get the required spacing \( s = 1.1 \text{ m} \). If, instead of a 125 mm \( \times \) 50 mm mandrel, a 150 mm \( \times \) 150 mm mandrel (\( r_{m,eq} = 84.63 \text{ mm} \)) is used for the installation of PVDs, then we have to use Fig. 7(a) (the curve corresponding to profile 3) to obtain \( n \). This gives \( n = 15.7 \). The equivalent unit cell radius \( r_{c,eq} \) corresponding to \( n = 15.7 \) is 0.52 m which yields a spacing \( s = 1.0 \text{ m} \). Note that, for the 150 mm \( \times \) 150 mm mandrel, the disturbed zones of adjacent unit cells overlap while, for the 125 mm \( \times \) 50 mm mandrel, no overlap occurs.

CONCLUSIONS

The effect of soil disturbance on the rate of PVD consolidation was studied using FEA maintaining the actual shapes of the PVD, disturbed zone and unit cell. For that purpose, a thorough characterization of the disturbed zone was done and it was found that the hydraulic conductivity profile in the disturbed zone can be expressed as a sigmoidal curve. A parametric study showed that the degree of disturbance in the immediate vicinity of the PVD and the extent of the disturbed zone affect the consolidation rate significantly. Proper quantification of the hydraulic conductivity profile in the disturbed zone is essential for an accurate prediction of the consolidation rate.

The PVD responses from the FEA were compared with those obtained with an analytical solution that assumes a bilinear variation for the hydraulic conductivity in the disturbed zone. The results matched reasonably well. Therefore, the analytical solution was adopted for producing design charts. The design charts can be used to obtain the PVD spacing, without any iteration, for a desired degree of consolidation within a specified time. The design charts can also be used for conditions in which overlapping of disturbed zones occurs.

REFERENCES


